Pre RMO 2013 Answers, Hints, Solutions, Analysis

Set –A

Q.1 (Q. 4 Set B)

Ans: 1

Hint: Pretty Straightforward Question

Solution: Write the given equation as $216k = a^n$. As 216 itself is a perfect power, i.e. $6^3, (a = 6, n = 3) \Rightarrow k = 1$

Analysis: The question only demands you to realize 216 is a perfect power of an integer. It could have been made more difficult if the question were $k(3^3 + 4^3 + 5^3 + 6^3) = a^n$or something of the sort. In that case, we would have $432k = a^n$. We might be able to guess the answer, but the idea is to prime factorize 432 as $2^4 3^3$, so for it to be a perfect power, $k = 3$. If it were $2^5 3^3 k = a^n$, then least value of k wouldn't be 3, but rather 2. It would be a good application of unique prime factorization of any natural number. One can generalize this. Many difficult questions are formed from simple ones with a little generalization. So in case you ever want to prepare seriously for Olympiads, try making new questions by looking for a simple generalization.

Q.2 (Q. 1 Set B)

Ans: 9

Hint: Rationalize the denominator and try writing the sum in expanded form. Do the same for the second Sum.

Solution: $S_n = \sum_0^n \frac{1}{\sqrt{k+1}+\sqrt{k}} = \sum_0^n \frac{\sqrt{k+1}-\sqrt{k}}{1} = (\sqrt{1}-\sqrt{0}) + (\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + \cdots (\sqrt{n+1}-\sqrt{n}) = \sqrt{n+1}$. Similarly, $\sum_1^{99} \frac{1}{S_n+S_{n-1}} = \sum_1^{99} \frac{1}{\sqrt{n+1}+\sqrt{n}} = \sqrt{100}-\sqrt{1} = 9$

Analysis: The question is a simple application of a very useful technique of finding sum of a sequence popularly known as the telescopic sum. In such a sum, to add a series, say $a_1 + a_2 + a_3 + \cdots + a_n$, try writing each term as difference of consecutive terms of some other sequence. i.e. writing $a_1 = b_2 - b_1, a_2 = b_3 - b_2, \dots, a_n = b_{n+1} - b_n$. In that case, the sum simply becomes $b_{n+1} - b_1$as the middle terms collapse just like the inner cylinders or a telescope leaving the first and the last term. Note that it may not always be possible to write a term as difference of two terms mentioned above. This makes it a difficult method to use in many questions. Such a technique therefore needs many skills like simplifying, factorizing, manipulating the expressions, observing some of the properties of the expression etc. But it is nonetheless a powerful method and in this particular question, the difference presents itself once the rationalization is done.

The problem here is similar to finding integral of a function. Given a function, we can always find its derivative, i.e. given a sequence we can always find the difference of consecutive terms, but the converse is not only difficult, but at times impossible. It is no wonder that the second fundamental theorem of calculus gives the result that looks familiar to the above sum (Compare $\sum_1^n a_n = b_{n+1} - b_n$with $\int_a^b f(x)\,dx = F(b) - F(a)$). A proof of the second fundamental theorem can be given in terms of a telescopic sum and the reader here is asked to explore the same.

Q.3 (Q. 3 Set B)

Ans: **0**

Hint: Note that if a_1 is one of such values of a, then $-a_1$is also one of the values. Alternately, try writing all the possible values of a by observing it is negative of a sum of two numbers whose product is 20.

Solution: Suppose α and β are the roots of the given equation. Therefore $\alpha\beta = 20$. Note that $-\alpha$ and $-\beta$ are also integers and $(-\alpha)(-\beta) = 20$. Meaning by changing the sign of a, we will get another set of integer solutions. Therefore sum of all such values of a will be zero as the terms will cancel in pairs.

Alternately, one may be interested in finding all the unordered pairs (α, β) satisfying the above, which happen to be $(20,1), (-20,-1), (10,2), (-10,-2), (5,4), (-5,-4)$, as we can factorize the number 20 only in these many possible ways. Therefore the possible values for $a = -(\alpha + \beta)$ are $21, -21, 12, -12, 9, -9$ the sum of whose is zero.

Analysis: It is a very good question on a combination of Number Theory and Algebra, many similar questions can be found in standard text. Although for this particular question, one can get the answer simply by observing the cancelation of positive and negative terms, The technique useful, In general, here is the fact that any integer can be factorized as a product of integers in a finite number of ways. By not neglecting the negative factors, one can get the combinations easily. Prime factorization again will play a role in writing the factors of the number. Feel free to modify this question so as to make it look more difficult but solvable under the above technique.

Q.4 (Q. 2 Set B)

Ans: **91**

Hint: Pretty Straight Forward.

Solution: It doesn't take much time to realize there are only two possibilities, $X - Z - Y$ or $Z - X - Y$. Note that $Y - Z - X$and $Y - X - Z$ are same as above (Why?). Then doing the necessary calculations, you may arrive at the desired answer.

Analysis: Questions like this are a bonus for anyone who just cannot get started in an exam as difficult as the Olympiad ones. It is a matter of psychology to conclude safely that anyone who has had a good start in any exam, is more likely to do well later than the one having a bad start. So always be on a lookout for such questions in any exam and don't spend too much time on a question that looks be trick or lengthy at first look.

Q.5 (Q. 5 Set B)

Ans: 10

Hint: There are three combinations possible, RG, GB or BR. For each of the three cases, find the total number of cases.

Solution: For RG, GB and BR, there are $(n-1)n, n(n+1)$ and $(n+1)(n-1)$ possibilities respectively the sum of whose is 299, giving $n = 10$.

Analysis: Again a straightforward problem once the key idea hits that is the fundamental principle of counting.

Q.6 (Q. 8 Set B)

Ans: 18

Hint: What is the least possible value for the number of digits of a number whose sum of digits is 2013? What combination will give the least number?

Solution: The number of digits of the numbers N cannot be less than 224 in which case the sum will be less than $223 * 9 = 2007$. Moreover if the number has 224 digits, the first digit cannot be less than 6 as the sum of digits would be less than $6 + 223 * 9 = 2013$. Now if the first digit is 6, the rest 223 digits would have to be 9 giving the sum of digits as 2013 and as there is no number less than this number whose sum of digits can be 2013, the least number $N = 6999 \dots 9$, with 9 appearing 223 times. Therefore $5N + 2013 = 5 * 6999 \dots 9 + 2013 = 5 * (7000 \dots 0 - 1) + 2013 = 35000 \dots 0 + 2008 = 35000 \dots 2008$. Thus the required answer is $3 + 5 + 2 + 8 = 18$.

Analysis: There is nothing special about the number 2013 except that it is the current year. We could easily make the question by replacing 2013 by any year (AD) or by any positive integer. The key idea is to use the fact that the maximum possible sum of digits for a given number of digits or the minimum possible number of digits for a given sum of digits is obtained by using the digit 9. One then can proceed easily by dividing 2013 by 9, obtaining the quotient and the remainder which here happened to be 223 and 6 respecitvely.

Number theory questions are a commonplace in Olympiad type of exams. In order to solve tough questions, it is a good idea to be equipped with some basic results in this area like divisibility, prime factorization, GCD, LCM, results regarding equations with integer solutions etc. Although this question requires hardly any theory, you may not be so lucky every time.

Q.7 (Q. 6 Set B)

Ans: **12**

Hint: n will be the LCM of 3 and 4.

Solution: A straightforward problem on the concept of Least Common Multiple.

Analysis: The essence of the problem is LCM and the story of Akbar Birbal only adds to the taste of the problem. Lookout for such questions as most of them are easier to solve and understand rather than some abstract unfamiliar question.

Q.8 (Q. 9 Set B)

Ans: **2**

Hint: First prove PQ is parallel to the parallel sides and then conclude its length from the midpoint theorem.

Solution: Let R be the midpoint of AB. From midpoint theorem, $PR||BC$ and $PR = \frac{1}{2}BC$. Similarly $QR||AD||BC||PR$ and $QR = \frac{1}{2}AD$. Thus the points P, Q and R are collinear. Thus $PQ = |PR - QR| = \frac{1}{2}|BC - AD| = 2$.

Analysis: A direct question from our question bank (Q. 44) luckily whose answers also match! A good application of mid-point theorem. The answer here can be guessed from intuition, so a clever student will get it without having to prove it. Nevertheless, such a guesswork are many times needed in order to solve a question. In case one has no idea where to start, make a guess, assume something and work in that direction. If you conclude something obvious, then work backwards and deduce what is needed. This approach is many times useful especially in proving inequalities. There is nothing wrong in doing so as long as all the steps are equivalent. In case you need to square or multiply somewhere, there may be loss or gain of additional information giving extra solutions or missing some, but these can mostly be countered. However, in this particular question, there's no need to justify as there's no place for justifying the answer, so do the justification only if you've some time left.

Q.9 (Q. 10 Set B)

Ans: $\mathbf{120^0}$

Hint: Find the angle BHC and BIC which should be equal in this question, then find the angle BOC in terms of the angle A.

Solution: As $BH \perp AC, \angle HBC = 90 - C$. Similarly, $\angle HCB = 90 - B$, thus $\angle BHC = 180 - A$. Similarly $\angle BIC = 180 - \frac{B}{2} - \frac{C}{2} = 90 + \frac{A}{2} = \angle BHC = 180 - A$, thus $A = 60^0$.
So $\angle BOC = 2A = 120^0$

Analysis: The question is a good example of problem based on Angle Chasing technique in Geometry where on expresses all the angles in terms of some known angle by using standard results likes corresponding/alternate angles, exterior angle theorems, angle in the same/alternate segment theorem, Using cyclic quadrilaterals etc. Many of the geometry problem become easier to handle if a student has done some homework of finding different angles in any triangle in terms of the angles of the triangle. There are many variations possible here like instead of saying B, H, I, C concyclic, say B, I, O, C are concyclic and so on. It is then just a matter of using the data in the given question to conclude the angles. The reader here is asked to play around different configurations possible in a triangle like Circumcircle, incircle, excircle configurations and try deducing as many angles possible in the figure as possible. This definitely pays off as some standard results and techniques become available.

Sadly one can get the answer by assuming the triangle to be an equilateral one without having to justify the same. Although this is in favour of all the students, it is not justifying in a qualification exam like this. But given this is only the primary stage for an Olympiad, it is not going to reflect in the final standing. So anyone seriously thinking of the Olympiads, must get a justification for any guesswork made in the paper afterwards.

Q.10 (Q. 7 Set B)

Ans: Bonus (Although the answer under some conditions is **3**)

Hint: Make a guesswork.

Solution: The question has infinite solutions. But if it is modified for non-negative integers, then one can get the answer for minimum possible sum of three numbers as follows. Assume $x > y \geq z \geq 0$ to be the three numbers without loss of generality. Then we get $x + yz = xy + xz$. For $y = 0$, one can conclude the answer. For $y \neq 0$, we have $y < x \Rightarrow yz \leq xz$ keeping in mind equality may occur if only if $z = 0$ as otherwise z is positive. Similarly $x \leq xy$ keeping in mind equality occurs if and only iff $y = 1$, therefore $yz + x \leq xz + xy = x + yz$. Thus equality must occur in all inequalities giving $z = 0, y = 1$ and $x = 2$. Thus $x + y + z = 3$.

Analysis: The problem is a good illustration of simple inequalities which are also a commonplace in Olympiads. In order to solve some equality problems, one may try proving that the equalities occur on boundaries, i.e. equality occurs when the max value of something equals the least of something.

In case the question stands as stated, it has infinite solutions one such type can be checked easily by taking $(x, y, z) = (t^2, t, -t)$ for some $t > 1$. Even for Non-negative numbers, the solutions are infinite as by any values of y and z between $\frac{1}{2}$ and 1so that $x = \frac{yz}{y+z-1} > 1 > y, z$ for all such y and z. Moreover if the condition for non-negative integers is added, it yet has infinite solutions as can be checked by putting $z = 0$ and $y = 1$ and x being any natural number greater than 1. So the question should have included three conditions in form of non-negative, integer numbers sum of whose is the least.

Q.11 (Q. 15 Set B)

Ans: 14

Hint: Add the three equations and then simplify by making perfect squares.

Solution: Adding the three equations yield $(x+1)^2 + (y+3)^2 + (z+2)^2 = 0$ which can be true iff $(x, y, z) = (-1, -3, -2)$. Thus giving $x^2 + y^2 + z^2 = 14$

Analysis: A direct question based on a question in our Pre RMO Selection Test (Q. 27) luckily whose answers also match! Another question on making use of inequalities to get a solution. Sum of perfect squares cannot be negative, a fact which is fundamental to most inequalities. Thus we have $(x+1)^2 + (y+3)^2 + (z+2)^2 \geq 0$, with equality occurring iff $(x, y, z) = (-1, -3, -2)$. Sadly one may again guess this solution by surpassing the cream of the problem.

One may try to use the values of y and z in one equation plugging the same in another and finally getting one equation in x. Although it is doable in theory, it is not a practical approach. One needs to be very much careful therefore in judging a question. The most obvious method may not be the most efficient and giving it up at right time would save precious time in the exam. It is therefore fruitful in many places where you're not able to solve a particular question at home as you know which approaches would not work. This is against the common belief where a student usually gets demotivated by not being able to solve a problem. But the fact remains that one learns much more by trying to solve a question and failing than by solving some question with ease. The path of success must pass through the experience of failure!

Q.12 (Q. 12 Set B)

Ans: 14

Hint: Easy question.

Solution: $PS = 2\sqrt{7}, BQ = PS = RC = 2\sqrt{7}$. Now use Pythagoras's theorem to the ΔPQC to get $PC = 14$.

Analysis: This paper would've been incomplete without some problem on the Pythagoras's theorem. Although such problems are plenty and rich, the current one happens to be easy.

Q.13 (Q. 13 Set B)

Ans: 8

Hint: Deduce what types of numbers will have same colour. Find total number of types to get maximum no. of colours needed.

Solution: The numbers having same colour as 1 are $14, 29, 44, \ldots 989$ and $16, 31, 46, \ldots 991$ i.e. the numbers of the form $15k \pm 1$ have same colour. Similarly the numbers of the form $15k \pm 2$ have same colour and so on till the numbers of the form $15k \pm 7$ requiring maximum total 7 different colours. Now finally numbers of the form $15k$ all have same colour. Thus maximum 8 colours are required.

Analysis: One important thing to note before even attempting to solve the question is that, the converse of the statement that numbers whose sum is divisible by 15 have same colours is not necessarily true. i.e. if two numbers have same colour, then their sum may or may not be divisible by 15. All the numbers are of the form $15k + r$, where $0 \leq r < 15$ and k being an integer. So it is safe to conclude that the numbers of the form $15k \pm r$ have same colours. Therefore r can take values from 0 to 7 giving 8 colours.

Q.14 (Q. 11 Set B)

Ans: 9

Hint: Try trial and error method.

Solution: $m = 1$ and $n = 8$.

Analysis: Typical number theory questions involving equations with integer answers either usually don't have a solution or have only trivial general solutions or have non-trivial solutions and usually all these cases are not so easy to arrive at. The question at hand only requires to find the least of all, which is easy to arrive by guessing. The general case may prove to be difficult or impossible however, i.e. to find all integers n where $1 + 2 + 3 + \cdots + n$ is a perfect square. Generalizations in number theory often leads to seemingly easy but difficult or almost impossible to prove problems. The reader here is advised to be aware of this particular aspect of Number theory and stay refrained from such generalization unless someone has solved it in fair amount of time or you're interested in doing research.

Q.15 (Q. 14 Set B)

Ans: 208

Hint: Conclude first that the quadrilateral formed by joining midpoints of any quadrilateral is always a parallelogram. Then in this particular question, conclude on such parallelogram is a rhombus.

Solution: As A_1B_1 and C_1D_1 are parallel and half of one of the diagonals of $ABCD$, (Why?) The quadrilateral $A_1B_1C_1D_1$ is a parallelogram. Similarly $A_2B_2C_2D_2$ is a parallelogram. As it is also given to be a rectangle, the diagonals of $A_1B_1C_1D_1$ are perpendicular to each other with lengths 8 and 12 (Why?). Thus it is a rhombus of side length $\sqrt{52}$. Thus the lengths of diagonals of $ABCD$ is $2\sqrt{52}$ each, giving the required product as 208.

Analysis: A question similar to the one in our question bank (Q. 40). A very good question on the application of mid-point theorem. This one result in Geometry has many applications including one of the proofs for concurrency of medians, co-linearity of Orthocentre-Centroid-Circumcentre. The Pythagoras's theorem is also hidden in the same. Some of the properties of special quadrilaterals such as parallelogram, rhombus and rectangle also come handy, so one must be very thorough with all these.

Q.16 (Q. 19 Set B)

Ans: 0

Hint: A question similar to question Q. 3. Use the same hint there.

Solution: Note that if α is a common root of the two equations for $b = b_1(say)$, then $-\alpha$ is the common root for $b = -b_1$. Thus all the pairs cancel each other to give final answer zero.

Analysis: Again a good question which can be done with little justification. It would be useful to investigate the question further to actually find all such values of b. The reader here is invited to do the same. There are only 3 such values.

Q.17 (Q. 18 Set B)

Ans: 60

Hint: Draw a clear diagram. Identify a triangle with sides $r, 200 - r$ and 100 with one angle 120^0 with r being the radius of S_1.

Solution: Denote the centres of the circles S_1 and S_2 by $O_1$and O_2 and radius of $S_1$by r. Radius of S is 200 (Why?). $O_2X = 200 \Rightarrow OX = 100$. Now $OO_1 = 200 - r$. So draw a separate ΔOXO_1, in which $\angle OXO_1 = 120^0, OX = 100, XO_1 = r$ and $OO_1 = 200 - r$. Now r can be obtained either directly by using cosine rule or by following. Drop perpendicular from O to O_1X extended to meet it in Z say. Therefore $OZ = 50\sqrt{3}, XZ = 50$ (Why?). Therefore in the right angle $\Delta OZO_1, (200 - r)^2 = (50 + r)^2 + \left(50\sqrt{3}\right)^2$ to get $r = 60$.

Analysis: This was apparently the most formidable question which even takes significant amount of time even to understand fully. Mostly one needs to draw the diagram several times to get it right. Such question are best kept at the end of the paper as chances are that one would spend significant amount of time without getting any conclusion. The key idea is a clear figure, some basic properties of circles touching each other and off course the Pythagoras's theorem ($30 - 60 - 90$ triangle is just its application). One more key idea is to draw a separate diagram once some of the features of the old diagram are used and there's no scope for further constructions so as to have some insight into the problem.

There is however a scope for confusion in the language of the problem where it doesn't mention which diameter the point X lies on, although it can be safely assumed as the diameter of S for the question to make sense.

Q.18 (Q. 17 Set B)

Ans: 61

Hint: Write 2013 as a sum of AP with common difference 1 then factorize one side of the expression.

Solution: Suppose a be the first integer in the n consecutive integers. We get $n(2a+n-1) = 4026 = 2 \cdot 3 \cdot 11 \cdot 61$. Now as $2a+n-1 > n \Rightarrow 4026 > n^2$, the maximum value of n occurs at $n = 61$.

Analysis: A very good question as a combination of number theory as well as inequalities. Factorization and on both sides as product of integers is the key which is a commonplace in many number theory questions involving integers. This is not the fact for non-integer numbers, i.e. for real numbers x and y, there are infinite solutions for $xy = n$, but for integers x, y and n, there are finite solutions for x and y.

There was a correction in the paper that included the term for positive integers. Even when the positive term is dropped, there is a simple solution. The reader here again is invited to find the same.

Q.19 (Q. 16 Set B)

Ans: **36**

Hint: Divide the area of the quadrilateral formed in two triangles. Now find the ratios of areas of the triangle thus formed to get the ratio of $CY : AY$. Then use Pythagoras's Theorem to ΔBCE to get the lengths.

Solution: Join BY and denote $A(\Delta \mathrm{AYX}) = A(\Delta BYX)(Why?) = x$ and $A(\Delta BCE) = y$. Thus $\frac{y+x}{y+2x} = \frac{13}{18} \Rightarrow \frac{y}{x} = \frac{8}{5}$. Therefore $\frac{CY}{AY} = \frac{A(\Delta \mathrm{BCY})}{A(\Delta \mathrm{BAY})} = \frac{y}{2x} = \frac{4}{5}$. Let $CY = 4t$, therefore $AY = BY = 5t$. Using Pythagoras's theorem in ΔBCE, we get $t = 4$. Thus $AC = 9t = 36$.

Analysis: A very good problem based on the concept that the ratio of areas of triangle with same height/base is same as the ratio of their bases/heights. This is a very useful fact in geometry that has far reaching consequences, one of which is the Cava's Theorem. The reader here is advised to explore the same. Pythagoras's Theorem is again critical in giving the exact answer. Also note how the calculation is made simpler by making use of variables for the areas and lengths. Such simplifications come in handy in geometry in order not to get lost in the long notations for lengths and areas which a student usually is familiar with. Using algebraic simplifications in geometry are essential in order to stay tuned with the essence of a good geometrical problem.

Q.20 (Q. 20 Set B)

Ans: **630**

Hint: The binary numbers (base 2) which are less than 64 in base 10, have at most 6 digits, three of which are 1 and the rest three as 0. 1 appears in every place ${}^5C_2 = 10$ number of times.

Solution: (Refer to Analysis part of you're not familiar with binary system) Denote n_2 for the binary representation, i.e. $10_2 = 2$ etc. A typical number satisfying the given conditions in the question is $001101_2 = 13$. There are ${}^6C_3 = 15$ such numbers less than 64. There are

${}^5C_2 = 10$ numbers with 1 in their unit place (Why?) whose place value is 1; ${}^5C_2 = 10$ numbers with 1 in the second place whose place value is $10_2 = 2$; ${}^5C_2 = 10$ numbers with 1 in the third place whose place value is $100_2 = 4$ and so on. Therefore the binary addition of all these numbers is the binary addition of all the place values of all these numbers (Why?) which is $10 * 1_2 + 10 * 10_2 + 10 * 100_2 + 10 * 1000_2 + 10 * 10000_2 + 10 * 100000_2 = 10 * (1 + 2 + 4 + 8 + 16 + 32) = 10 * 63 = 630$.

Analysis: The usual decimal representation makes use of ten different digits $0, 1, 2, \dots, 9$ for writing down a number. There's nothing special to the number ten here except that we all have ten fingers to count. We might as well have had 8 digits to represent all numbers (0 to 7: leading to octal system) or 16 digits (0 to 9 and A to F: leading to Hexadecimal system) or simply two digits (0 and 1: Binary system, i.e. base 2). Just like $1, 10, 100, 1000$ ...denote the powers of 10 in decimal system, $1_2, 10_2, 100_2, 1000_2, \dots$ denote the powers of 2 in binary system, i.e. $1_2 = 1, 10_2 = 2, 100_2 = 4, 1000_2 = 8, \dots$and so on. The other numbers are just the unique combinations of these numbers, i.e. $11_2 = 3, 101_2 = 5, 110_2 = 6, 111_2 = 7, \dots$. We can always convert from decimal to Binary and Vice versa as follows. To convert 1101001_2 into decimal, write it in terms of powers of 2 as $1101001_2 = 1 * 2^6 + 1 * 2^5 + 0 * 2^4 + 1 * 2^3 + 0 * 2^2 + 0 * 2^1 + 1 * 2^0 = 105$. The reader here is invited to decide on an algorithm to convert from decimal to binary. The rest of the problem can be solved with little knowledge of combinatorics.

Given that we all are too used to the decimal system till school level, the problem would present a challenge to first understand it in terms of the new language so defined. But once you're familiar with the system, the arithmetic is just as same as that for the decimal numbers. Although these problems are not a frequent place in Olympiads, there is hardly any challenge in mastering the basics. A keen pursuer of Olympiads is therefore advised to get comfortable with Binary, Octal and Hexadecimal system and play around with some

Overall Comments:

The paper seemed to have lived up to the expectations of being an Olympiad paper for the primary stage. Important topics like Geometry, Number Theory and Algebra have been given justice and although there are no direct questions on inequalities, there is hardly anything that suggests they have been ignored. Given the mindset of a student and his/her preparation, there's hardly any question making use of any sophisticated theory, although the questions may be a bit challenging at times. With the exception of Q. 10, there are no issues in the paper.

As far as the preparation of such an exam goes, there unfortunately seems hardly any options available. The only serious effort taken from the concerned authorities is after the INMO, i.e. the 2nd stage (3rd stage in case of Mumbai). Many gifted students might just miss this because of lack of preparation in the very early stage. Anyone who is aspiring for doing well in Math Olympiads, must start working from the schools itself, as during the Class XI/XII, a student is more focused on Engineering entrance exam preparations. It is not at all an impossible task as the syllabus for Olympiads is just pre-Calculus, i.e. School Maths till class X. A reader here is invited to explore the wonderful knowledge of pre-college mathematics by following some standard available but less popular literature.