INDIAN ASSOCIATION OF PHYSICS TEACHERS
NATIONAL STANDARD EXAMINATION IN PHYSICS 2007-2008
Total time : 120 minutes (A-1, A-2 & B)

PART - A **(Total Marks : 180)**

SUB-PART A-1

Q.1 The distance traveled by an object is given by $x = at + bt^2/(c + a)$ where t is time and a, b, c, are constants. The dimensions of b and c respectively are :
(A) $[L^2T^{3}]$, $[L\,T^{1}]$ (B) $[L\,T^{2}]$, $[L\,T^{1}]$
(C) $[LT^{1}]$, $[L^2\,T^{1}]$ (D) $[L\,T^{1}]$, $[L\,T^{2}]$

Sol. **[A]**
Use the dimensional analysis. Note that the dimensions of a and c are the same as those of [length/time] and those of b are [length × (length/time) $(\text{time})^2$].

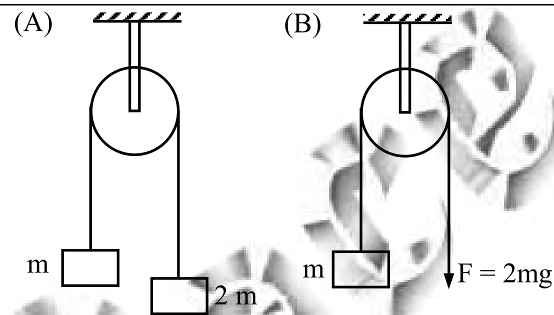
Q.2 A person throws vertically up n balls per second with the same velocity. He throws a ball whenever the previous one is at its highest point. The height to which the balls rise is :
(A) g/n^2 (B) 2gn (C) $g/2n^2$ (D) $2gn^2$

Sol. **[C]**
The time required for a ball to reach highest point is (1/n) second giving initial velocity to be (g/n). Use this in $v^2 = u^2 - 2\,gh$ to find h.

Q.3 A particle moves at a constant speed v from point A to point B along a circle of radius r. If points A and B have an angular separation of θ, the magnitude of change in velocity in moving from A to B is :
(A) zero (B) 2v sin (θ /2)
(C) 2v sin θ (D) 2v cos (θ/2)

Sol. **[B]**
The change in velocity is $[v_B - v_A] = [v_B = + (-v_A)]$. Find the magnitude nothing that the angle between v_B and v_A is (180º – θ).

Q.4 Two pulley arrangements (A) and (B) are as shown in the figure. Neglect the masses of the ropes and pulleys and the friction at the axle of the pulleys. The ratio of the acceleration of mass m in arrangement (A) to that in arrangement (B) is



(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 2 : 1

Sol. **[C]**
For arrangement (A), the acceleration of mass m is obviously [(2m – m)g/(2m + m)] = g/3. For arrangement (B), the net force acting on mass m is mg upwards so that the acceleration is g only and hence the result.

Q.5 A particle of mass m is made to move with uniform speed v along the perimeter of a regular hexagon. Magnitude of impulse applied at each corner is :
(A) mv (B) $mv\sqrt{3}$
(C) mv/2 (D) $mv/\sqrt{3}$

Sol. **[A]**
The impulse is nothing but change in momentum. For taking the difference of two momenta, note that the angle between vectors at the corner is 120º.

Q.6 The maximum tension in the string of a pendulum is three times the minimum tension. If θ_0 be the angular amplitude, cos θ_0 is
(A) 1/2 (B) 3/4
(C) 3/5 (D) 2/3

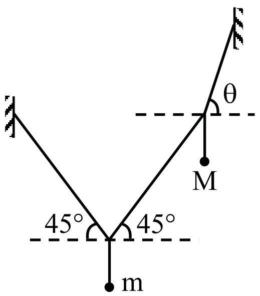
Sol. **[C]**
Tension in the string is maximum [mg(3 – 2 cosθ)] when it is in the vertical position whereas tension is minimum [mg cosθ] when the string is in the extreme position.

Q.7 A body of mass 4m at rest explodes into three fragments. Two of the fragments, each of mass m move with speed v in mutually perpendicular directions. Total kinetic energy released in the process is :

(A) mv^2 (B) $3mv^2/2$
(C) $2mv^2$ (D) $3mv^2$

Sol. **[B]**
Consider the conservation of linear momentum along two perpendicular directions X and Y axes, to get the velocity of the largest particle [(v/2)(– i – j)]. Then calculate the kinetic energy.

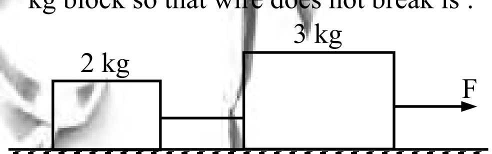
Q.8 Two masses m and M are attached to strings as shown in the figure. In equilibrium tan θ is



(A) 1 + (2M /m) (B) 1 + (2m /M)
(C) 1 + (M/2m) (D) 1 + (m/2M)

Sol. **[A]**
Use the concept of balancing components of forces and tensions along horizontal and vertical directions at the two points where the masses m and M are attached.

Q.9 Two bodies of masses 2 kg and 3 kg are connected by a metal wire of cross section 0.04 mm^2. Breaking stress of metal wire is 2.5 GPa. The maximum force F that can be applied to 3 kg block so that wire does not break is :



(A) 100 N (B) 150 N (C) 200 N (D) 250 N

Sol. **[D]**
If T is the tension in the string connecting the two bodies, (T/A) ≤ the breaking stress, where A is the area of cross section. Deduce the relation T = (2/5) F and then the result.

Q.10 A ball floats on mercury in a container with volume V_1 inside mercury. If the container is now covered and the air inside is pumped out, volume V_2 is found to be under mercury. Then,

(A) $V_1 = V_2$ (B) $V_1 > V_2$
(C) $V_2 > V_1$ (D) $V_2 = 0$

Sol. **[C]**
Since air is pumped out, upthrust due to air becomes zero and the ball sinks slightly more than before.

Q.11 Pressure of one litre of nitrogen (γ = 1.4) is 500 cm of mercury. It is compressed adiabatically to 990 cc. The final pressure of the gas (in cm of mercury) is :

(A) 507 (B) 505
(C) 495 (D) 502

Sol. **[A]**
Use the expression for adiabatic change to get (dp/p) + y (dV/V). After substintution dp = 7. Note that there is an increase of pressure.

Q.12 Two identical rings A and B are acted upon by torques τ_A and τ_B respectively. A is rotating about an axis passing through the centre of mass and perpendicular to the plane of the ring. B is rotating about a chord at a distance $(1\sqrt{2})$ times the radius of the ring. If the angular acceleration of the rings is the same, then

(A) $\tau_A = \tau_B$
(B) $\tau_A > \tau_B$
(C) $\tau_A < \tau_B$
(D) Nothing can be said about τ_A and τ_B as data are insufficient

Sol. **[A]**
Using perpendicular and parallel axes theorems it is found that the moments of inertia in both the cases are the same.

Q.13 Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hour respectively. The radius of the orbit of S_1 is 10^4km. When S_1 is closest to S_2, the angular speed of S_2 as observed by an astronaut in S_1 is

(A) π rad/hr (B) π/3 rad/hr
(C) 2π rad/hr (D) π/2 rad/hr

Sol. **[B]**

Use Kepler's law to get the radius of orbit of S_2 to be 4×10^4 km. The linear speeds of S_1 and S_2 happen to be $2\pi \times 10^4$ km/hr and $\pi \times 10^4$ km/hr. Then, the angular speed of S_2 as seen from S_1 is $(\Delta v/\Delta r)$.

Q.14 If a section of a soap bubble of radius r by a plane through its centre is considered, the force on the half due to surface tension is :

(A) 2π rT (B) 4π rT

(C) $\pi r^2 T$ (D) 2r T

Sol. **[B]**

Note that force = 2 (2π r) T.

Q.15 Volume of a gas ($C_p/C_V = \gamma$) expands from a volume V to 2V at constant pressure p. The change in internal energy is :

(A) $R / (\gamma - 1)$ (B) pV

(C) $pV / (\gamma - 1)$ (D) γ pV $(\gamma - 1)$

Sol. **[C]**

Use the expression for the change in internal energy = nC_V dT. Also use the substitution $C_V = R / (\gamma - 1)$ and pV/T = constant.

Q.16 A satellite is revolving round the earth with orbital speed v_0. If it imagined to stop suddenly, the speed with which it will strike the surface of the earth would be (v_e – escape velocity of a body from earth's surface)

(A) v_e^2 / v_0 (B) v_0

(C) $(v_e^2 - v_0^2)^{1/2}$ (D) $(v_e^2 - 2 v_0^2)^{1/2}$

Sol. **[D]**

Use the concept of conservation of energy and the expressions for escape velocity and orbital velocity.

Q.17 A car moves with a speed of 60 km /hr from point A to point B and then with the speed of 40 km / hr from point B to point C. Further it moves to a point D with a speed equal to its average speed between A and C. Points A, B, C and D are collinear and equidistant. The average speed of the car between A and D is :

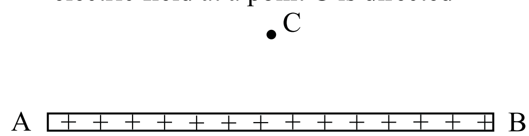
(A) 30 km/ hr (B) 50 km / hr

(C) 48 km / hr (D) 60 km / hr

Sol. **[C]**

Noting that AB = BC, average speed between A and C is 48 km/hr and that between A and D is also 48 km/hr.

Q.18 A long thin rod AB is charged uniformly. The electric field at a point C is directed



(A) parallel to the rod

(B) perpendicular to the rod

(C) along the bisector of the angle ACB

(D) along a line dividing the angle ACB in the ratio BC : AC

Sol. **[C]**

Check it by considering two elements at the two ends then two symmetrically situated elements of the rod.

Q.19 An electric field is given by E = $y\hat{i} + x\hat{j}$ volt/m. The work done in moving a charge of $10\mu C$ from a point $r_1 = 3\hat{i} + 4\hat{j}$ to another $r_2 = 2\hat{i} + 6\hat{j}$ is :

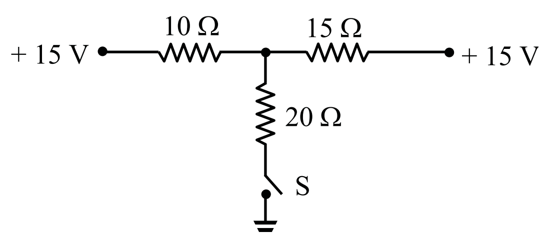
(A) $10\sqrt{5}$ J (B) $-10\sqrt{5}$ J

(C) $10\sqrt{2} \times 10^{-6}$ J (D) zero

Sol. **[D]**

Use the relation : work done = charge × potential difference. Then, the potential difference ($-\int E \cdot dr$) comes out to be negative integral of (y dx + x dy), that is, of [d (xy)] to be evaluated between(3,4) and (2, 6).

Q.20 When the switch S is closed in the circuit shown below, the current that flows through it is



(A) zero (B) 15/26 A

(C) 15/13 A (D) 5/26 A

Sol. **[B]**

Take the potential at the junction of resistors to be V and then use Kirchhoff' s current law at this junction. Obtain V and then the current through 20 ohm resistor, that is, the switch.

Q.21 In a standing wave formed as a result of reflection from a surface, the ratio of the amplitude at an antinode to that at node is x. The fraction of energy that is reflected is :

(A) $[(x - 1) / x]^2$ (B) $[x / (x + 1)]^2$
(C) $[(x - 1)/(x + 1)]^2$ (D) $[1/x]^2$

Sol. **[C]**
If a_i and a_r denote the amplitudes of the incident and the reflected waves then the net amplitude at the antinode is $(a_i + a_r)$ and that at the node is $(a_i - a_r)$. From this we get $(a_r / a_i) = [(x - 1)/(x + 1)]$. Note that the energy is proportional to square of the amplitude.

Q.22 The fundamental frequency of a sonometer wire of length l is n_0. A bridge is now introduced at a distance of Δl ($<< l$) from the centre of the wire. The lengths of wire on the two sides of the bridge are now vibrated in their fundamental modes. Then, the beat frequency nearly is :

(A) $n_0 \Delta l / l$ (B) $8 n_0 \Delta l / l$
(C) $2 n_0 \Delta l / l$ (D) $n_0 \Delta l / 2l$

Sol. **[B]**
Note that the beat frequency is $(n_1 = n_2)$ and the corresponding vibrating lengths are $(l/2 - \Delta)$ and $(l/2 + \Delta l)$.

Q.23 Two open organ pipes of fundamental frequencies n_1 and n_2 are joined in series. The fundamental frequency of the new pipe so obtained will be :

(A) $n_1 + n_2$ (B) $n_1 n_2/ (n_1 + n_2)$
(C) $n_1 n_2/(n_1 - n_2)$ (D) $\sqrt{n_1^2 + n_2^2}$

Sol. **[C]**
Lengths of the organ pipes are $(v/2n_1)$ and $(v/2n_2)$ where v is the speed of sound in air. The fundamental frequency of the new organ pipe (after the two are joined) will be $[v/2(n_1 + n_2)]$.

Q.24 The specific heat of a solid at low temperature varies according to the relation $c = k T^3$ where k is a constant. The heat required to raise the temperature of a mass m of such a solid from T = 0 K to T = 20 K is

(A) 2×10^4 mk (B) 4×10^4 mk
(C) 8×10^4 mk (D) 16×10^4 mk

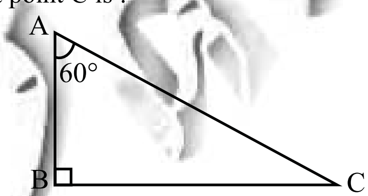
Sol. **[B]**
Integrate (mCdT) between 0 K and 20 K.

Q.25 In case of an adiabatic process the correct relation in terms of pressure p and density ρ of a gas is

(A) $p\rho^{\gamma}$ = constant (B) $p^{\gamma}\rho^{\gamma\ \ 1}$ constant
(C) $p\rho^{\gamma\ \ 1}$ = constant (D) $p\rho^{\ \gamma}$ = constant

Sol. **[D]**
Note that $_pV^{\gamma}$ = constant and that ρ is inversely proportional to V.

Q.26 Three rods of the same cross section and made of the same material form the sides of a triangle ABC as shown. The points A and B are maintained at temperatures T and $\sqrt{2}$T respectively in the steady state. Assuming that only heat conduction takes place, the temperature at point C is :



(A) $[(2\sqrt{2} + \sqrt{3})/(2 + \sqrt{3})]T$
(B) $[3\sqrt{2}/(2 + \sqrt{3})]T$
(C) $[2/\sqrt{3}]T$
(D) $[\sqrt{5}/2]T$

Sol. **[A]**
In s steady state, the heat conducted from B to C is the same as from C to A.

Q.27 A soap bubble filled with helium floats in air. Let m_w and m_{He} be the mass of the wall of the bubble and that of the helium gas inside, respectively. If density of helium is 0.1384 times that of air, then :

(A) $m_w > m_{He}$
(B) $m_w < m_{He}$
(C) $m_w = m_{He}$
(D) nothing can be said about m_w and m_{He} as data are insufficient

Sol. **[A]**
Density of helium is less than half the density of air and hence the mass of helium is less than half the mass of air. Note that $(m_w + m_{He}) \leq m_{air}$.

Q.28 A ball A moving with certain velocity in positive X axis direction collides with a stationary ball B. After collision their directions of motion make angles α and β with the X axis. The possible values of α and β are

(A) $\alpha = 30°$ and $\beta = -45°$

(B) $\alpha = 90°$ and $\beta = -120°$

(C) $\alpha = 0°$ and $\beta = -30°$

(D) $\alpha = 45°$ and $\beta = 0°$

Sol. **[A]**
This is the only possibility where the condition for conservation of momentum can be satisfied.

Q.29 Two identical thin planoconvex lenses of refractive index n are silvered, one on the plane side and the other on the convex side. The ratio of their focal lengths is :

(A) n / (n – 1) (B) (n – 1) / n

(C) (n + 1) / n (D) n

Sol. **[A]**
When the plane face of a planoconvex lens is coated, the focal length is given by R/[2(n – 1)] and it is R/2n when the convex face is coated.

Q.30 Two plane mirrors subtend angle θ between them. A ray of light incident parallel to one of them retraces its path after n reflections. The graph of θ (Y axis) versus n (X axis) is :

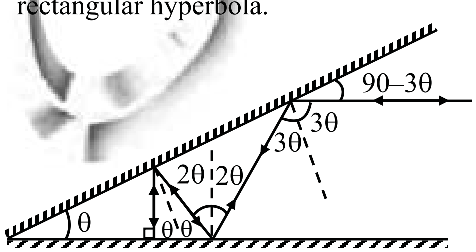
(A) a straight line through origin

(B) a parabola

(C) a rectangular hyperbola

(D) a straight line with a intercept on Y axis

Sol. **[C]**
Refer to the ray diagram and note that for four reflections from the two mirrors, we have $90° - 3\theta = \theta$, so that $\theta = 90°/4 = 90°$/(number of reflections n). Therefore, the product of θ and n is a constant so that the graph is a rectangular hyperbola.



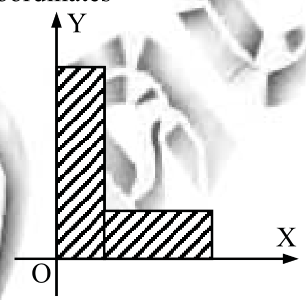
Q.31 A charge + 6μC is situated inside a closed surface. The flux, in volt-m, through a portion of the surface subtending a solid angle of (π/2) at the point where the charge is situated is

(A) $+ (1.5/\varepsilon_0) \times 10^{6}$ (B) $+ (6/\varepsilon_0) \times 10^{6}$

(C) $- (1.5/\varepsilon_0) \times 10^{6}$ (D) $+ (0.75/\varepsilon_0) \times 10^{6}$

Sol. **[A]**
Note that the flux will be $[(\pi/2)/4\pi] (q/\varepsilon_0)$

Q.32 A metal sheet 14 cm × 2cm of uniform thickness is cut into two pieces of width two cm. The two pieces are joined and laid along XY plane as shown. The centre of mass has the coordinates



(A) (1, 1) (B) (19/7, 19/7)

(C) (8/7, 8/7) (D) (12/7, 12/7)

Sol. **[B]**
Note that the centres of mass of the vertical and the horizontal pieces are (1, 4) and (5, 1) respectively and their masses are in the ratio 4 : 3.

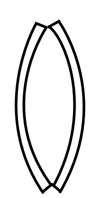
Q.33 In a double slit experiment, the wavelength of monochromatic light used is λ and the distance between the slits is d. The screen is at a distance D from the slits. If a bright fringe is formed opposite to a slit on the screen, the order of the fringe is :

(A) d/2D (B) $d^2/\lambda D$

(C) $d^2/2\lambda D$ (D) $\lambda D/d^2$

Sol. **[C]**
The distance from the centre of the screen is d/2 = nX where X is the fringe width and n is the order of the fringe.

Q.34 A lens formed by two watch glasses, as shown, behaves like a

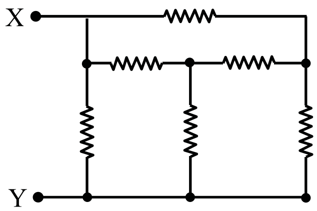


(A) convex lens (B) concave lens

(C) glass plate (D) mirror

Sol. **[C]**
Note that for each of the watch glasses, the two radii of curvature happen to be the same so that their focal lengths happen to be infinite.

Q.35 Six resistors each of 10 ohm are connected as shown. The equivalent resistance between points X and Y is :



(A) 20 ohm (B) 5 ohm
(C) 25/3 ohm (D) 10 ohm

Sol. **[B]**
Redrawing the circuit reveals that five of the resistors form a balanced Wheatstone's network between points X and Y with the remaining resistor in parallel appearing across X and Y.

Q.36 Two long parallel straight conductors carry currents i_1 and i_2 ($i_1 > i_2$).When the currents are in the same direction, the magnetic field at a point midway between the wires is 20 μT. If the direction of i_2 is reversed, the field becomes 50 μT. The ratio of the currents i_1/i_2 is :
(A) 5/2 (B) 7/3 (C) 4/3 (D) 5/3

Sol. **[B]**
Note that when the currents are in the sense the magnetic fields due the two wires subtract and when the currents are in opposite sense they add.

Q.37 Magnetic field at the centre of a circular loop of area A is B. The magnetic moment of the loop is :
(A) $BA^2/(\mu_0\pi)$ (B) $BA\sqrt{A}/\mu_0$
(C) $BA\sqrt{A}/(\mu_0\pi)$ (D) $2BA\sqrt{A}/(\mu_0\sqrt{\pi})$

Sol. **[D]**
Use the expression for the magnetic field at the centre of a circular coil [$B = (\mu_0 i) / 2r$]and that for the magnetic moment [M = iA]. Note that area $A = \pi r^2$.

Q.38 A current of 1A through a coil of inductance of 200 mH is increasing at a rate of 0.5 A/s. The energy stored in the inductor per second is :
(A) 0.5 J/s (B) 5.0 J/s
(C) 0.1 J/s (D) 2.0 J/s

Sol. **[C]**
Note that the emf induced in the inductor is [L (di/dt)] and energy stored per unit time is the power, that is, [emf × current]

Q.39 According to Bohr theory, for the stability of an atom, angular momentum of an electron in an orbit is quantized. This is a necessary condition according to :
(A) Pauli's exclusion principle
(B) the concept of wave associated with an electron
(C) correspondence principle
(D) none of the above

Sol. **[B]**
Note that angular momentum mvr = n(h/2 π) giving n [h/(mv)] = 2 π r. Note that [h/(mv)] is the de Broglie wavelength.

Q.40 A radioactive substance with decay constant of 0.5/s is being produced at a constant rate of 50 nuclei per second. If there are no nuclei present initially, the time (in second) after which 25 nuclei will be present is :
(A) 1 (B) 2 ln (4/3)
(C) ln 2 (D) ln (4/3)

Sol. **[B]**
Use the relation di/dt = 50 – λN and integrate to get $N = 50/\lambda\ [1 - e^{\lambda t}]$. Then use λ = 0.5/s and N = 25.

Sub-Part A-2

Q.41 Two particles having the same specific charge enter a uniform magnetic field with the same speed but at angles of 30° and 60° with the field. Let a, b, and c be the ratios of their periods, radii and pitches of their helical paths respectively, then
(A) abc > 1 (B) $a + b = 2\sqrt{C}$
(C) $a^2 = C$ (D) a b = C

Sol. **[A,B,C,D]**
Use the expression for periodic time T = (2π m) /(Bq sin θ), for radius of the circular path r = (mv)/(Bq sin θ) and for the pitch of the helical path p = T (v cos θ) where the symbols their usual meanings. With this we get, $a = b = \sqrt{3}$ and c = 3.

Q.42 Let v and a bet he instantaneous velocity and the acceleration respectively of a particle moving in a plane. The rate of change of speed (dv/dt) of the particle is :

(A) | a |

(B) (v · a)/ | v |

(C) the component of a in the direction of v

(D) the component of a perpendicular to v

Sol. **[B,C]**

Use $v^2 = v_X^2 + v_y^2$, differentiate to get (dv/dt) = $(v_x a_x + v_y a_y)/ v$ = (v · a)/ v. Also note that (v/v) is the unit vector along v.

Q.43 A piece of metal weighs 210 g in air, 180 g in water and 120 g in a liquid. Then, specific gravity of :

(A) metal is 3 (B) metal is 7

(C) liquid is 3 (D) liquid is 1/3

Sol. **[B,C]**

Use relation : relative density of metal = $W_{air}/(W_{air} - W_{water})$ and loss of weight in liquid = upthrust in liquid.

Q.44 Two sphere A and B have the same radii but the heat capacity of A is greater than that of B. The surfaces of both are painted black. They are heated to the same temperature and allowed to cool in vacuum. Then,

(A) A cools faster than B

(B) both A and B cool at the same rate

(C) at any temperature the ratio of their rates of cooling is a constant

(D) B cools faster than A

Sol. **[C,D]**

At any temperature θ, both the spheres lose heat at the same rate which is C (dθ/dt) in general.

Q.45 Two different coils have self inductance L_1 = 10 mH and L_2 = 12 mH. the currents in both the coils are increased at the same rate and the power in the two coils is also the same at some instant of time. At that instant of time let i_1, V_1 and W_1 be the current, the induced emf and the energy stored respectively in the first coil. Let i_2, V_2 and W_2 be the corresponding quantities for the second coil. Then,

(A) $i_1/i_2 = 6/5$ (B) $i_1/i_2 = 5/6$

(C) $W_2/W_1 = 5/6$ (D) $V_2/V_1 = 6/5$

Sol. **[A,C,D]**

Use the relation power P = Vi = L(di/dt) i. Since P and (di/dt) are the same, $L_1i_1 = L_2i_2$. Note that the energy stored in an inductor is $(1/2\ Li^2)$.

Q.46 A ball A moving with a velocity 5 m/s collides elastically with another identical ball at rest such that the velocity of A makes an angle of 30° with the line joining the centres of the balls. Then,

(A) speed of A after collision is (5/2) m/s

(B) speed of B after collision is $(5\sqrt{3}/2)$ m/s

(C) balls A and B move at right angles after collision

(D) Kinetic energy is not conserved as the collision is not head-on

Sol. **[A,B,C]**

Consider the conservation of kinetic energy and the fact that the momentum must be conserved along two directions, namely (i) line joining their centres and, (ii) perpendicular to that line.

Q.47 For any monoatomic gas the quantity/quantities independent of the nature of the gas at the same temperature is /are

(A) the number of molecules is one mole

(B) the number of molecules in equal volume

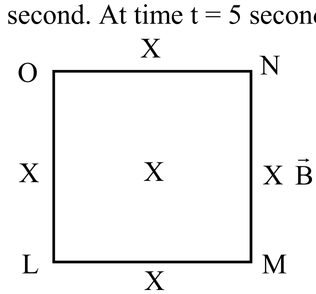
(C) the translational kinetic energy of one mole

(D) the kinetic energy of unit mass

Sol. **[A,C]**

Consider the properties of one mole of any gas at a given temperature.

Q.48 A square coil OLMN of side 5 cm is placed in a magnetic field $B = 3kt^2$ (as shown in the figure) where k is a constant, B is in tesla and t is in second. At time t = 5 second



(A) induced current flows from O to L

(B) induced current flows from L to O

(C) induced emf is 75k mV

(D) induced emf is 1.875 k V

Sol. **[A,C]**

Use the expression : emf induced = dϕ/dt = l^2 (dB/dt) and consider the sense in which the induced emf sends the current.

Q.49 Two card A and B are moving in the same direction with speeds 36 km/hr and 54 km/hr respectively. Car B is ahead of A. If A sounds horn of frequency 1000 Hz and the speed of sound in air is 340 m/s, the frequency of sound received by the driver of car B is

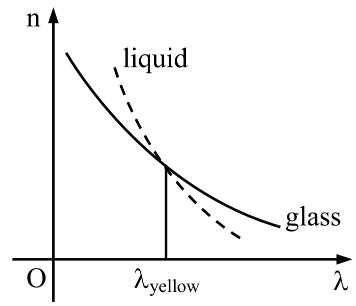
(A) 928.57 Hz (B) 984.84 Hz

(C) 946.37 Hz (D) 938.47 Hz

Sol. **[B]**

Noting that speed of the observer is 15 m/s and that of the source is 10 m/s apply Doppler's relation for the apparent frequency.

Q.50 A glass prism is immersed in a hypothetical liquid. The curves showing the refractive index n as a function of wavelength λ for glass and liquid are as shown in the figure. When a ray of white light is incident on the prism parallel to the base –



(A) yellow ray travels without deviation

(B) blue ray is deviated towards the vertex

(C) red ray is deviated towards the base

(D) there is no dispersion

Sol. **[A,B,C]**

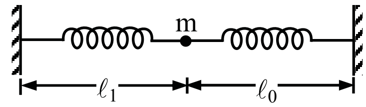
Note that the refractive indices of both the glass and the liquid are the same for yellow and hence no deviation. However, red ray enters from a rarer to a relatively denser medium while blue ray enters from a denser medium to a relatively rarer medium and hence the corresponding deviations.

PART B Marks : 60

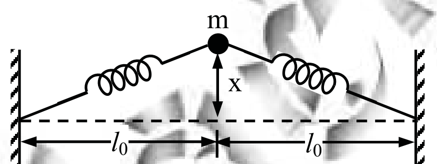
*** All questions are compulsory.**

*** All questions carry equal marks**

Q.51 A small bob joins two light unstretched identical springs fixed at their far ends and arranged along a straight line, as shown in the figure. The bob is displaced in a direction perpendicular to the line of the springs by x ($<<I_0$) and then released. Obtain an expression for the acceleration of the bob in terms of the displacement x. Is the motion simple harmonic ?



Sol. The length of the stretched spring $l = (l_0^2 + x^2)^{1/2} \approx l_0 + x^2\ (2l_0)$ so that the extension of the spring is $(l - l_0) = x^2 / (2l_0)$. Now the tension in the spring will be T = k $[x^2/(2l_0)]$. The restoring force will be 2 T sin $\theta = kx^3 / l_0^2$ where $\sin\theta \approx \theta = x/l$. This gives the acceleration proportional to x^3. The motion is obviously not simple harmonic.



Q.52 A body of mass m is projected inside a liquid at an angle θ_0 with horizontal at an initial velocity v_0. If the liquid develops a velocity dependent force F = – kv where k is a positive constant, determine the x and the y components of the velocity at any instant. Also determine its range.

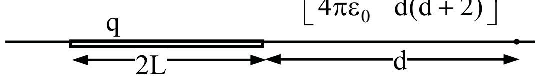
Sol. To determine the component of velocity in the horizontal direction, consider the equation $F_x = -k\,v_x$ and integrate to get ln v = – (kt/m) + ln C where the constant of integration C can be determined by using the initial condition that at t = 0, $v_x = v_0 \cos\theta_0$. This gives after substitution $v_x = v_0 \cos\theta_0\ (e^{\,kt/m})$. Similarly the vertical component of velocity can be determined by considering the equation $F_y = -kv_y - mg$. While integrating we use the condition that at t = 0, $v_y = v_0 \sin\theta_0$. This gives $v_y = (m/k)\ \{[k/m)\ v_0 \sin\theta_0 + g]\ e^{\,kt/m} - g\}$. The range can then be determined by writing $F_x = -k\,v_x$ as $m\,(dv_x dt) = -k\,v_x$ and further as $mv_x(dv_x dx) = -k\,v_x$. this given the range $x_{max} = (m\,v_0 \cos\theta_0)/k$.

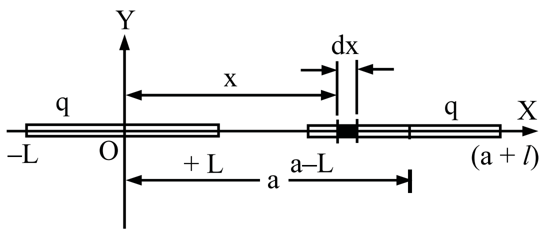
Q.53 Identical thin rods of length 2*l* carry equal charges + q, uniformly distributed along their lengths. The rods lie along X axis with their centres separated by a distance of a > 2*l*. Show that the magnitude of the force exerted by one rod on the other is given by

$F = (1/4\pi\varepsilon_0)\ (q^2/4l^2)\ \ln\,[a^2/(a^2 - 4\,l^2)]$

Sol. Referring to the figure, write the electric field at a point distance d from one end of the rod. This comes out to be [$(1/4\pi\varepsilon_0)q/\{d(d + l)\}$]. Using this write the electric field dE at a point distance x from the origin where the centre of one of the rods is situated. This is given by $dE = (1/4\pi\varepsilon_0)\, q/[(x - l)(x + l)]$. From this, the force dF on a small element of charge λdx of the other rod can be written as ($dE \times dq$). integrating this relation between ($a - l$) and ($a + l$) we get the result.

$$E = \left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{d(d+2)}\right]$$

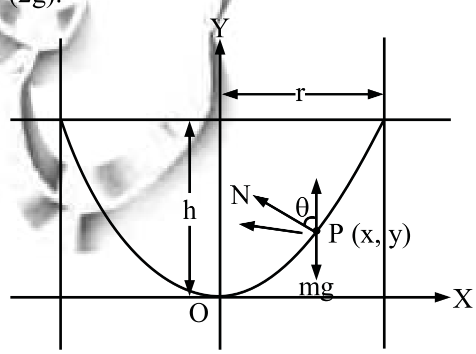




Q.54 A liquid is kept in a cylindrical vessel which is rotating about its axis, as a result of which the liquid rises at the sides. Show that the section of the surface of the liquid by a vertical plane containing the axis is a parabola. Determine the difference in height of the liquid at the centre of the vessel and its sides.

Sol. Referring to the figure, consider a particle at a point P on the surface. We have $N \cos\theta = m g$ and $N \sin\theta = m x \omega^2$, giving $\tan\theta = (x \omega^2) / g$. The Slope of the curve (dy/dx) is itself $\tan\theta$. With this we get the differential equation $(dy/dx) = (x \omega^2) / g$. Solve this differential equation using the conditions that at $x = 0$, $y = 0$ and at $x = r$, $y = h$. This gives $h = (\omega^2 r^2) / (2g)$.

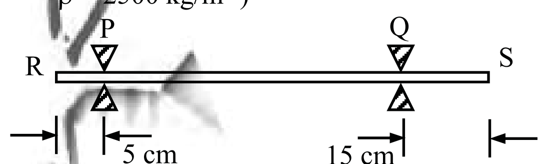
section by a vertical plane



Q.55 A container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27°C and pressure of 1×10^5 N/m^2. The total mass of the mixture is 28g. If the molecular weights of neon and argon are 20 and 40 respectively, determine the masses of the individual gases in the mixture, assuming them to be ideal. (R = 8.314 J/mole K)

Sol. Let m_1 and m_2 be the masses of neon and argon respectively. Then, $(m_1 + m_2) = 28$. The number of moles of the two gases are (m_1/M_1) and (m_2/M_2) where M_1 and M_2 are the corresponding molecular weights. Using $pV = nRT$ where n represents the total number of moles, we get $m_1 = 4$ g and $m_2 = 24$g

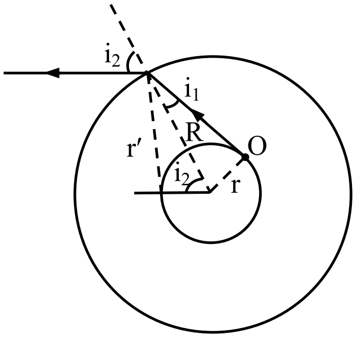
Q.56 A metal rod of length 1 m is clamped at two points as shown in the figure. Find the minimum frequency of natural longitudinal oscillation of the rod. (Young's modulus $Y = 1.6 \times 10^{11}$ N/m^2, density of metal $\rho = 2500$ kg/m^3)



Sol. Using the relation $v = (Y/\rho)^{1/2}$ the speed of longitudinal wave is 8000 m/s. Nodes will be formed at the clamped positions and there should be integral number of loops between the nodes. If a denotes the number of loops between P and Q, then $a(\lambda/2) = 80$ or $a\lambda = 160$. Now, since R and S are free ends, the number of loops between P-R and Q-S must be odd multiples of ($\lambda/4$) say, b and c respectively. This gives $(2b - 1)(\lambda/4) = 5$ and $(2c - 1)(\lambda/4) = 15$. For frequency to be minimum, a, b and c must be smallest integers. With this a = 8, b = 1 and c = 3, and then, frequency comes to be 40 kHz.

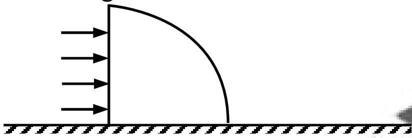
Q.57 The mercury thread in a glass thermometer appears to be half as thick as the cylindrical stem. Calculate the actual diameter o the mercury thread if the actual diameter of the stem is 3mm. Refractive index of glass is 1.5. How does the answer depend upon the external diameter ? Draw a next ray diagram.

Sol. Refer to the ray diagram and write $\sin i_1 = r/R$ and $\sin i_2 = r'/R$. This gives the refractive index $n = \sin i_2 / \sin i_1 = r' / r$ = (apparent radius) / (actual radius) giving actual radius as 1 mm. the answer is obviously independent of the external diameter R.

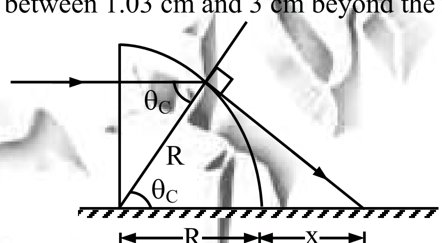


$r \rightarrow$ actual radius
$r' \rightarrow$ apparent radius
$O \rightarrow$ Point from where a ray starts

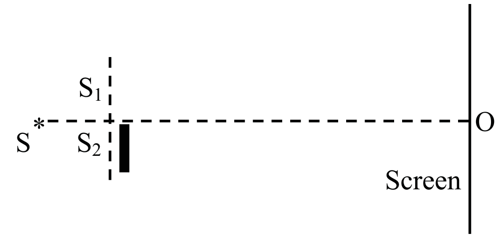
Q.58 A glass prism in the shape of a quarter cylinder lies on a horizontal table. A horizontal beam of light falls on its vertical plane surface, as shown. If the radius R of they cylinder is 3 cm and the refractive index n of the glass is 1.5, where on the table beyond the cylinder will a patch of light be formed ?



Sol. Referring to the ray diagram, we have, $\sin \theta_c = 1/n$ and $\cos \theta_c = R/(R + x)$ giving x = 1.03 cm. Now consider the lower part of the cylinder as a planoconvex lens to get f = 2R = 6. cm. Thus the path of light will be found between 1.03 cm and 3 cm beyond the cylinder.



Q.59 The Young's double slit experiment is done in water of refractive index 4/3. A light source of wavelength 6000Å is used and the slits are 0.45 mm apart. One of the slits is covered by a glass plate of thickness 10.4 μm and refractive index 3/2. The interference pattern is observed on a screen placed 1.5 m from the slits. Determine (1) the location of the central maximum, and (2) the intensity of light at point O relative to the maximum intensity.



Sol. Let the central maximum be formed at P at a distance x from O. Then, the path difference $\{S_2P + [(\mu_g/\mu_m) - 1]\, t\} - S_1P = 0$, where t is the thickness of the glass plate and μ_g, μ_m are the refractive indices of the glass of the plate and the medium respectively. This gives x = 4.33 mm. The path difference for waves reaching at point O is $[(\mu_g / \mu_m) - 1]\, t$ which corresponds to a phase difference of $(13\pi/3)$. If I_0 denotes the intensity at point O and I_{max} the maximum intensity, then the ratio $I_0/I_{max} = 3/4$. Note that the net intensity I at any point due to two waves with intensities I_1 and I_2 is $[I_1 + I_2 + 2\,(I_1I_2)^{1/2} \cos \delta]$, where δ is the phase difference.

Q.60 It is proposed to use the nuclear reaction ${}_{84}Po^{210} \rightarrow {}_{82}Pb^{206} + {}_2He^4$ to produce 2kW of electric power in a generator. The half life of polonium (${}_{84}Po^{210}$) is 138.6 days. Assuming efficiency of the generator to be 10%, calculate how many grams of polonium are required per day.
[Masses of nuclei : Po^{210} – 209.98264 amu, Pb^{206} – 205.97440 amu, He^4 – 4.00260 amu and 1 amu = 931 MeV]

Sol. The mass difference (Δm) between the two isotopes is 0.00564 amu which is equivalent to 5.25 MeV. The decay constant λ turns out to be (0.693/138.6) = 0.005 /day. If M grams of Po^{210} are required per day, the number of nuclei in M grams is about $(6 \times 10^{23}/210)$ M = N, say. Then, λN should be the number of nuclei present. The energy produced per day will then be λN $(5.25 \text{ MeV} \times 1.6 \times 10^{19})$ J. This energy is expected to be (2kW × 24 × 60 × 60) J. Equating the two we get M = 14.4 gram. Since the efficiency is 10%, the amount required is 144 gram.